WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMADSE03T-MATHEMATICS (DSE1/2)

PROBABILITY AND STATISTICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Give axiomatic definition of probability.

(b) Consider an experiment of rolling two dice. Define a random variable over the event space of this experiment.

(c) Consider an experiment of tossing two coins. Find the probability of two heads given atleast one head.

(d) Prove that $P(B\,|\,A)\geq 1-\dfrac{P(\overline{B})}{P(A)}$

(e) Distribution function $F(x)$ of a random variable X is given by

$$F(x) = 1-\frac{1}{2}e^{-x},\ x\geq 0$$
$$= 0,\ \text{elsewhere}$$

Find $P(X=0)$ and $P(X>1)$.

(f) The joint density function of X and Y is given by

$$f(x,y)=\begin{cases} k(x+y), & x>0,\ y>0,\ x+y<2 \\ 0 & \text{elsewhere}\end{cases}$$

Find the value of k.

(g) Two random variables X and Y have zero means and standard deviations 1 and 2 respectively. Find the variance of $X+Y$ if X and Y are uncorrelated.

(h) State Tchebycheff's inequality.

(i) Explain what are meant by a statistic and its sampling distribution.

2. (a) Two cards are drawn from a well-shuffled pack. Find the probability that at least one of them is a spade. 3+5

(b) Obtain the Poisson approximation to the binomial law, on stating the assumption made by you.

3. A random variable X has the following probability distribution: 2+(3+3)

x_i	0	1	2	3	4	5	6	7
f_i	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(a) Find k

(b) Evaluate $P(X<6)$, $P(X \geq 6)$

4. (a) Let $f(x, y)$ be the joint p.d.f. of X and Y. Prove that X and Y are independent if and only if $f(x, y) = f_x(x)\ f_y(y)$. 4+4

(b) Write down the distribution function $\phi(x)$ of a standard normal distribution and prove that $\phi(0) = \frac{1}{2}$.

5. (a) If X be a $\gamma(l)$ variate, find $E\{\sqrt{X}\}$ 2+(3+3)

(b) Find the mean and standard deviation of a binomial distribution.

6. (a) The joint density function of X and Y is given by 5+3

$$f(x, y) = \begin{cases} k(x+y), \text{ for } 0<x<1, 0<y<1 \\ 0, \quad \text{elsewhere} \end{cases}$$

Find

(i) the value of k

(ii) the marginal density functions

(iii) the conditional density functions

Are X and Y independent?

(b) The joint density function of the random variable X, Y is given by:

$$f(x, y) = 2\ (0<x<1, 0<y<x).$$

Compute $P\left(\frac{1}{4} < X < \frac{3}{4} \,\middle|\, Y = \frac{1}{2}\right)$

7. (a) If σ_x^2, σ_y^2 and σ_{x-y}^2 be the variances of X, Y and $X-Y$ respectively, then prove that 3+3+2

$$\rho_{xy} = (\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2)/2\sigma_x\sigma_y .$$

(b) Find k such that $\rho_{uv} = 0$ where $U = X + kY$ and $V = X + \frac{\sigma_x}{\sigma_y}Y$.

(c) If one of the regression coefficients is more than unity, prove that the other must have been less than unity.

8. (a) Define the concept of convergence in probability. If $X_n \xrightarrow[\text{in } p]{} X$, $Y_n \xrightarrow[\text{in } p]{} Y$, as $n \to \infty$, show that $X_n \pm Y_n \xrightarrow[\text{in } p]{} X \pm Y$ as $n \to \infty$ 5+3

(b) State Central Limit Theorem for independent and identically distributed random variable with finite variance.

9. Let $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)$ be the stationary distribution for a Markov Chain on the state space $\{1, 2, 3, 4\}$ with transition probability matrix *P*. Suppose that the states 1 and 2 are transient and the states 3 and 4 form a communicating class. Which of the following are true? 8

(a) $\mu p^3 = \mu p^5$

(b) $\mu_1 = 0$ and $\mu_2 = 0$

(c) $\mu_3 + \mu_4 = 1$

(d) One of μ_3 and μ_4 is zero.

OR

(a) Prove that Central Limit Theorem (for equal components) implies Law of Large Numbers for equal components. 5+3

(b) A random variable X has probability density function $12x^2(1-x), (0 < x < 1)$. Compute $P(|X - m| \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality.

10.(a) Find the maximum likelihood estimate of σ^2 for a normal (m, σ) population if *m* is known. 4+4

(b) The wages of a factory's workers are assumed to be normally distributed with mean *m* and variance 25. A random sample of 25 workers gives the total wages equal to 1250 units. Test the hypothesis $m = 52$ against the alternative $m = 49$ at 1% level of significance.

$$\left[\frac{1}{2\pi}\int_{-\infty}^{2.32} e^{-\frac{x^2}{2}}\,dx = 0.01\right]$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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