WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMADSE05T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Give an example of an order preserving map between two ordered sets.

(b) In any lattice L, prove that

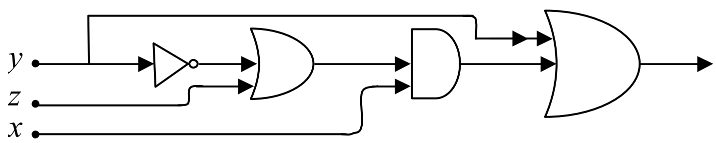
$$x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z),$$

for all $x, y, z \in L$.

(c) Use a Karnaugh-map to find the minimized sum-of-product Boolean expression of the Boolean expression

$$xyz + xyz' + xy'z' + x'yz + x'yz'.$$

(d) Write down the Boolean expression that represents the following logic-circuit.



(e) On the alphabet $\Sigma = \{0, 1\}$, show that

$$1^*0 + 1^*0(\lambda + 0 + 1)^*(\lambda + 0 + 1) = 1^*0(0 + 1)^*$$

where λ is the empty string over Σ.

(f) Give state diagram of a DFA recognizing the following language over the alphabet $\{0, 1\} : \{w \mid w$ is any string except 11 and 111$\}$.

(g) Draw a derivation tree that yields $a^4 \in L(G)$, where $G = (\{s\}, \{a, b\}, S, P)$ is a context-free grammar with $P = \{S \to ss, S \to a\}$.

(h) Can a Turing machine contain just a single state? Give reasons.

2. (a) Define maximal and minimal elements in a poset. 1

(b) Show that any finite nonempty subset X of a poset has minimal and maximal elements. 2

(c) Let $(P, \leq)$ be a finite poset. Show that the order $\leq$ can always be extended to a total order $\preceq$ on P, in the sense that, for all $x, y \in P$, $x \leq y \Rightarrow x \preceq y$. 2

(d) Using the result stated in (c), determine two total ordering relations on the set of positive divisors of 36 into which the order of the poset D_{36} of divisors of 36 can be extended. 3

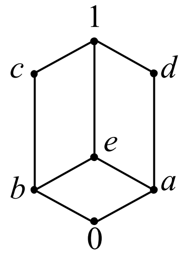
3. (a) For two lattices L and K, prove that a mapping $\phi : L \to K$ is a lattice isomorphism if only if ϕ is an order isomorphism. 4

(b) In any lattice L, prove that the following identities are equivalent: 2

(i) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \ \forall x, y, z \in L$

(ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c), \ \forall a, b, c \in L$.

(c) The Hasse diagram given below represents a lattice: 2



Is this lattice distributive? Justify your answer with proper reason.

4. (a) Define a modular lattice. 1

(b) Show that every distributive lattice is modular. 1

(c) Draw the Hasse diagram for a pentagon N_5 of five elements. Show that the lattice N_5 is non-modular. 3

(d) Let L be a lattice such that none of its sublattices is isomorphic to a pentagon. Prove that L is a modular lattice. 3

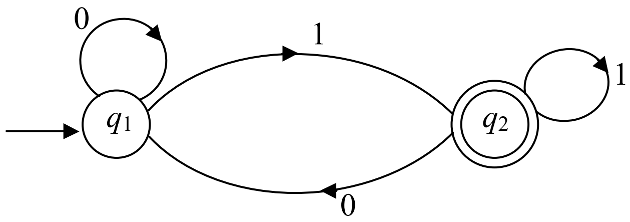
5. (a) Find the essential prime implicants of the Boolean function $f(A, B, C, D) = \sum m(1, 5, 6, 12, 13, 14)$. Hence find the minimal expression for $f(A, B, C, D)$ by using Quine-McClusky method. 3+3

(b) Find the Boolean expression in CNF which generates the following truth function: 2

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	1

6. (a) Let $R \subseteq \Sigma^*$ and $\lambda \notin R$; where λ is the empty string. For any $S \subseteq \Sigma^*$, prove that $S = SR$ if and only if $S = \phi$. 2+2

(b) Consider the binary alphabet $\Sigma = \{0, 1\}$. Determine the regular expression for the language recognized by the DFA, M whose transition graph is as follows: 4



7. (a) Use the pumping lemma for context free languages to show that the language $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free. 4

(b) Let $M = (S, \Sigma, \delta, q_0, F)$ be a non-deterministic finite automaton, in which $S = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $F = \{q_2\}$ and the transition function δ is given by the following transition table: 4

δ	0	1
$\to q_0$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ

Construct a three-state DFA, M_1 equivalent to NFA, M. Also draw the transition graph of the DFA, M_1.

8. (a) Define Chomsky normal form of context-free grammar. 1

(b) Transform the grammar with productions 3

$S \to aSaaA \mid A$

$A \to abA \mid bb$

into Chomsky normal form.

(c) Let L_1 be a context-free language and L_2 be a regular language. Prove that $L_1 \cap L_2$ is a context-free language. 4

9. Show that the collection of (Turing) decidable languages is closed under the operation of 2+2+2+2

(i) union

(ii) concatenation

(iii) star

(iv) complementation.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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