WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMADSE06T-MATHEMATICS (DSE3/4)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

   (a) Find the equation of the line of action of the resultant of a system of coplanar forces having total moments G, $2G$, $3G$ about the points $(0, 0), (0, 1), (2, 4)$ respectively.

   (b) What is meant by limiting friction? Why is it limiting?

   (c) Find the centre of gravity of a surface revolving round the axis of y.

   (d) Define Pointsot's central axis of a system of forces acting on a body.

   (e) Are the centre of suspension and centre of oscillation of a compound pendulum reversible? Justify your answer.

   (f) Find the degrees of freedom of three particles in a two-dimensional plane, two of which are connected by a fixed straight line.

   (g) Define an apse and apsidal angle for a central orbit.

   (h) What is the difference between a simple pendulum and a compound pendulum?

UNIT-I

ANALYTICAL STATICS

2. (a) Find the condition for the astatic equilibrium of a rigid body acted on by a system of coplanar forces. 4

   (b) A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Find the position of the centre of gravity of the remainder. 4

3. Forces P, Q, R act along three straight lines given by the equation $y = 0$, $z = c$; $z = 0$, $x = a$; $x = 0$, $y = b$. Find the pitch of the equivalent wrench. 4+4

   Also show that if the wrench reduces to a single force, then the line of action of the forces lies on the hyperboloid $(x - a).(y - b).(z - c) = xyz$.

4. (a) A solid frustum of paraboloid of height h and latus rectum $4a$, rests with its vertex on the vertex of a paraboloid of revolution of latus rectum $4b$. Deduce the condition of stable equilibrium of the system. 4

(b) Two equal uniform rods AB and AC, each of length $2l$ are freely jointed at A and rest on a smooth vertical circle of radius 'a'. Show that if the angle between the rods be $\frac{\pi}{2}$, then $l = 2a$. 4

UNIT-II

ANALYTICAL DYNAMICS

5. Derive the components of velocity and acceleration of a particle referred to a set of rotating rectangular axes. 4+4

6. (a) Find the condition that the orbit of a satellite will be an ellipse, parabola or a hyperbola. 4

(b) A particle describes an ellipse under a force $\frac{\mu}{(\text{distance})^2}$ towards a focus. If it was projected with velocity V from a point at a distance r from the centre of force, show that the periodic time is 4

$$\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{V^2}{\mu}\right)^{-3/2}$$

7. (a) Deduce the differential equation of a central orbit under a central force in two-dimensional polar coordinates. 4

(b) A circular orbit of radius 'a' is described under the central attractive force $f(r)=\mu\left(\frac{b}{r^2}+\frac{c}{r^4}\right)$, $\mu>0$. Deduce the condition of stability of the motion. 4

8. (a) Define momental ellipsoid and find it at the centre of an elliptic plate. 4

(b) Deduce the equation of motion of a rigid body from D'Alembert's principle. 4

9. (a) Show that the moment of inertia of elliptic area of mass M and semi axes a and b about a diameter of length r is $\frac{M}{4}.\frac{a^2b^2}{r^2}$. 5

(b) A fine string has two masses M and M' tied to its ends and passes over a rough pulley, of mass m whose centre is fixed, if the string does slip over the pulley, show that M will descend with acceleration $\frac{M-M'}{M+M'+mk^2/a^2}.g$ where a is the radius and k is the radius of gyration of the pulley. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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